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TECHNIQUES FOR THE DERIVATION AND EVALUATION OF TRUNCATED GRAVITATIONAL MODELS FOR SPECIAL APPLICATIONS

George T. Stentz, et al

Defense Mapping Agency Aerospace Center
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GEORGE T. STENTZ
HASCHAL L. WHITE
SPACE SCIENCE BRANCH
GEOPHYSICAL AND SPACE SCIENCES DIVISION
RESEARCH DEPARTMENT

DEFENSE MAPPING AGENCY
AEROSPACE CENTER
ST. LOUIS AFS, MISSOURI 63118

PREFACE

GENERAL: This publication is one of a series of Technical Reports related to the fields of mapping, charting, and geodesy, and their related arts and sciences. Each Technical Report is issued to disseminate results of studies performed by Defense Mapping Agency Aerospace Center personnel.

PURPOSE: This report is issued to present to interested organizations and individuals the results of a study to determine the best approach for the derivation of truncated gravitational models. Three methods of derivation at two levels of truncation along with evaluation results are presented.

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REVIEWED

William P. Wall

WILLIAM P. WALL
Asst Chief, Research Department

Lawrence F. Ayers

LAWRENCE F. AYERS
Technical Director

APPROVED

Donald D. Hawkins

DONALD D. HAWKINS, Colonel, USAF
Director

ABSTRACT

In many applications, it is necessary to use a truncated gravitational model because of limitations in computer hardware and software or mission requirements. Consequently, studies have been performed to evaluate the effect of truncation and also to determine the best method for the derivation of eighth and 12th degree and order gravitational models. Three methods were used for their derivation; truncation by chopping the original more complete model, suppression of parameter corrections for all harmonic coefficients above the desired degree and order, and by not allowing as parameters any gravitational model coefficients above the desired degree and order limitation. The procedures for the derivation and subsequent analysis were applied to both the Department of Defense World Geodetic System 1972 and the 1969 Smithsonian Standard Earth (II) gravitational models.

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INTRODUCTION

The Department of Defense World Geodetic System 1972 (WGS 72) [1] gravitational model consists of zonal harmonic coefficients through the 24th degree and tesseral harmonics complete through degree and order 19. An additional 37 pairs of resonant harmonics, most of which are 12th through 16th order, are also included. This results in a total of 473 non-zero gravitational model coefficients. This is a relatively large gravitational model that may exceed limitations of computer hardware and software or mission requirements. This study is directed toward evaluating the best method to use for the derivation of an eighth and also a 12th degree and order truncated gravitational model under the assumption that such a model is required for some applications.

DISCUSSION

1. Techniques of Derivation

Three different techniques for the derivation of truncated spherical harmonic coefficient models of the earth's gravity field were investigated. These methods were applied to WGS 72 and the Smithsonian Astrophysical Observatory Standard Earth (II), SAO SE (II), [2] gravitational models. The SAO SE (II) model was chosen to be used to verify results obtained from WGS 72 because both models are similar in that they contain satellite and surface gravity data.

a. Chopping Method

The first method of truncation will be referred to as chopping. Chopping is applied directly to the WGS 72 and SAO SE (II) gravitational model coefficients by simply ignoring the existence of coefficients above the eighth and also the 12th degree and order. This results in the coefficients of these two truncated models having the same numerical value as their counterparts in the original model. When this approach is used, the contribution of coefficients above the degree of truncation is lost.

b. Suppression of Parameters Method

The second method, suppression of parameters, involves the solution of a suppressed normal matrix (N) for parameter corrections ($\underline{\Delta P}$) to the initial gravitational model parameter set ($\underline{P}_0$). The following equations illustrate the procedure,

$$N\underline{\Delta P} = \underline{b}$$

$$\underline{\Delta P} = N^{-1}\,\underline{b}$$

$$\underline{P} = \underline{P}_0 + \underline{\Delta P}$$

where $\underline{P}$ is the new parameter set (gravity model), $\underline{b}$ the discrepancy vector or right hand side, and N^{-1} the inverse normal matrix. This method results in zero corrections ($\Delta P=0$) above the desired level of truncation because the rows and columns pertaining to the higher degree parameters are eliminated from the normal equations.

The reduced set of equations is then compressed to its lesser rank and solved. This procedure produces a gravitational model of "improved" coefficients up through degree eight or 12, depending on the level of truncation, with the initial gravitational model coefficients for the higher degree terms. The effect of this approach is similar to the preceding method except that suppression of parameters eliminates the contribution of the higher degree non-zero initial rather than the corrected coefficients. This amounts to the elimination of a significant number of coefficients for the SAO SE (II) model since it has initial estimates for all of its coefficients. The WGS 72 model has initial estimates for all coefficients below and for 19 pairs above the 12th degree and order. Table 1 identifies the non-zero initial parameters above the 12th degree and order for both SAO SE (II) and WGS 72.

c. Adjustment of Parameters Method

In the adjustment of parameters method, the right hand side ($\underline{b}$) is adjusted so that the solution obtained represents the complete gravitational model. That is, the adjustment results in a zero value for all initial gravitational model coefficients above the selected level of truncation. This is accomplished by adjusting the right hand side for a parameter difference ($\underline{\Delta P}_a$) which is defined by the equation,

$$\underline{\Delta P}_a = \underline{P}_n - \underline{P}_o$$

Table 1

Non-Zero Starting Parameters Above the Twelveth Degree (n) and Order (m) for SAO SE (II) and WGS 72 Gravitational Models

SAO SE (II)			WGS 72
n,m	n,m	n,m	n,m
13,1	14,12	16,7	13,12
13,2	14,13	16,8	13,13
13,3	14,14	16,9	14,11
13,4	15,1	16,10	14,12
13,5	15,2	16,11	14,13
13,6	15,3	16,12	14,14
13,7	15,4	16,13	15,12
13,8	15,5	16,14	15,15
13,9	15,6	16,15	16,12
13,10	15,7	16,16	16,15
13,11	15,8	17,12	17,12
13,12	15,9	17,13	17,14
13,13	15,10	17,14	18,12
14,1	15,11	18,12	18,14
14,2	15,12	18,13	19,13
14,3	15,13	18,14	19,14
14,4	15,14	19,12	20,13
14,5	15,15	19,13	21,13
14,6	16,1	19,14	22,13
14,7	16,2	20,13	
14,8	16,3	20,14	
14,9	16,4	21,13	
14,10	16,5	21,14	
14,11	16,6	22,13	
		22,14	

In this equation, $\underline{P}_n$ represents the new and $\underline{P}_o$ the old initial parameter sets. For the special case under consideration, $\underline{\Delta P}_a$ is equal to zero below ($\underline{P}_n = \underline{P}_o$) and $-\underline{P}_o$ above ($\underline{P}_n = 0$) the degree and order limits selected for the gravitational model to be derived. The new parameter corrections after the adjustment ($\underline{\Delta P}_n$) are defined by the equation,

$$\underline{\Delta P}_n = \underline{P} - \underline{P}_n ,$$

and the total parameter corrections ($\underline{\Delta P}$) are represented by the sum of $\underline{\Delta P}_a$ and $\underline{\Delta P}_n$. That is,

$$\underline{\Delta P} = \underline{\Delta P}_n + \underline{\Delta P}_a = \underline{P} - \underline{P}_n + \underline{P}_n - \underline{P}_o = \underline{P} - \underline{P}_o .$$

Upon substituting the above equation for $\underline{\Delta P}$ into the equation

$$N\underline{\Delta P} = \underline{b}$$

it becomes

$$N(\underline{\Delta P}_n + \underline{\Delta P}_a) = \underline{b}$$

and

$$N\underline{\Delta P}_n = \underline{b} - N\underline{\Delta P}_a .$$

After the adjustment of the right-hand side ($\underline{b}$) by the $-N\underline{\Delta P}_a$, the normal matrix is suppressed and a solution made. Least squares solutions were made using this technique to obtain an eighth and also a 12th degree and order gravitational model. This approach would appear to be the one to use for a limited gravitational model because

the adjustment process should result in the absorption of as much of the gravitational field as possible into the derived model coefficients and thus minimize the effect of truncation by omission as in the two preceeding methods.

2. Evaluation of Models

Three methods were used to evaluate the derived truncated gravitational models. They are computation and comparison of degree variances, comparison of mean gravity anomalies computed from the gravitational model with those developed from observed surface gravity data, and the use of the gravitational model in different orbit reduction applications followed by an evaluation of residuals and orbital differences. These methods were used as an aid in the development of the WGS 72 and are merely extended here to evaluate the various truncated models.

a. Comparison of Degree Variances

The computation and comparison of degree variances can be used as an indicator of the validity of a gravitational model. Various authors; Kaula [3], Rapp [4], and Pellinen [5], have estimated the magnitude of degree variances. These estimates serve as a criteria to establish bounds for the degree variances computed from the harmonic coefficient set. However, the question here is not so much the magnitude of the degree variances, but the changes in the coefficients obtained by the truncation methods as reflected through the degree variances. The degree variances of the SAO SE (II) and WGS 72 gravitational models truncated by these three methods; chopping, suppression, and

adjustment, are shown in Tables 2 and 3, respectively. The eighth and also the 12th degree models obtained by suppressing the higher degree and order parameter changes are in very close agreement with the chopped degree variances for the SAO SE (II) (Table 2) and similarly for the WGS 72 (Table 3). However, for the adjusted eighth degree models, there is an increase in the SAO SE (II) seventh and eighth as well as for the WGS 72 sixth and seventh degree variances. For the 12th degree models, the SAO SE (II) shows some changes in the degree variances below the eighth degree and significant changes for the nineth, 10th, 11th, and 12th degrees. Similar changes occurred for the 10th and 11th degree variances for WGS 72. These changes show a tendency of the adjusted gravitational models to absorb some of the residual effect resulting from the omission of some of the higher degree gravitational field variations. However, further tests and evaluations are necessary to determine which of the truncated gravitational models give the best gravity field representation.

b. Comparison with Observed Mean Free Air Surface Gravity Anomalies

Comparison of mean gravity anomalies computed from a gravitational model with those developed from observed surface gravity data is another method for evaluating gravitational models. In this test, the 5°x5° mean anomalies developed from surface gravity data are divided into three groups: all 5°x5°

Table 2

SAO SE (II) Degree Variances

Degree	Method of Derivation					
	Chopped	Suppressed	Adjusted	Chopped	Suppressed	Adjusted
2	7.4	7.4	7.8	7.4	7.4	8.0
3	33.0	29.5	30.7	33.0	29.4	30.2
4	20.0	19.3	17.7	20.0	19.3	18.0
5	17.8	17.9	14.3	17.8	17.8	14.7
6	15.7	15.5	12.2	15.7	15.4	12.5
7	15.5	15.7	22.4	15.5	15.6	23.1
8	6.7	6.5	20.0	6.7	6.6	14.9
9				12.7	12.7	53.0
10				12.9	12.0	87.2
11				12.8	15.9	81.3
12				5.1	8.1	52.9

Table 3

WGS 72 Degree Variances

Degree	Method of Derivation					
	Chopped	Suppressed	Adjusted	Chopped	Suppressed	Adjusted
2	7.6	7.7	7.6	7.6	7.6	7.7
3	34.3	34.4	35.0	34.3	34.3	34.8
4	19.0	19.5	19.6	19.0	19.3	19.5
5	20.8	20.4	20.0	20.8	20.3	19.9
6	19.7	21.7	27.3	19.7	21.4	25.4
7	21.2	21.6	32.5	21.2	21.6	27.9
8	11.1	13.5	14.3	11.1	11.8	11.4
9				10.2	10.4	12.0
10				9.3	10.6	23.1
11				7.3	10.6	45.2
12				4.7	6.1	11.7

mean anomalies for which at least 10 1°x1° mean anomalies were observed and representing 70 percent of the earth's surface (Group A, Table 4); all 5°x5° mean anomalies for which 20 or more 1°x1° mean anomalies were observed and representing 36 percent of the earth's surface (Group B, Table 4); and the 5°x5° mean anomalies with all 25 1°x1° mean anomalies observed and representing six percent of the earth's surface (Group C, Table 4). These data groups are indicative of one of the limits of this method of evaluation in that an increase in worldwide coverage results in a decrease in the percentage of 1°x1° observations in the 5°x5° mean anomaly data. However, the results of this comparison are shown in Table 4. This tabular data is the $\langle (g_T - g_S)^2 \rangle$ introduced by Kaula [6]. In this notation, $\langle (g_T - g_S)^2 \rangle$ is the mean square difference of g_T and g_S where g_T is the observed mean free air gravity anomaly and g_S is the mean anomaly computed from the gravitational model coefficients. Based on this analysis, all of the chopped models are superior to the adjusted models (smaller $\langle (g_T - g_S)^2 \rangle$) at both truncation levels for the WGS 72 and SAO SE (II) gravitational models. Improved results also occur when the suppressed and chopped models are truncated at the 12th rather than the eighth degree. The reverse is true for the adjusted models.

Table 4

Comparison of 5°x5° Equal Area Mean Gravity Anomalies Derived From Gravitational Model Coefficients With Those Computed From Terrestrial Data (Units=Mgals^2)

Gravitational Model	Derivation		Mean Square Difference $<(g_T-g_S)^2>$		
	Method	Degree	Group A	Group B	Group C
WGS 72	Chopped	8	159	170	123
	Suppressed	8	162	176	140
	Adjusted	8	181	196	137
	Chopped	12	137	139	79
	Suppressed	12	145	149	72
	Adjusted	12	235	234	124
SAO SE (II)	Chopped	8	158	174	119
	Suppressed	8	158	173	119
	Adjusted	8	202	223	152
	Chopped	12	146	154	88
	Suppressed	12	148	154	91
	Adjusted	12	406	345	121

c. Satellite Orbit Reductions

The evaluation of gravitational models for orbit reduction applications can be accomplished by performing orbit reductions followed by orbital comparisons and data residual analysis. However, there are several factors to be considered when interpreting the results. Some of these are; only the gravitational model coefficients that produce detectable perturbations are evaluated, resonant harmonic coefficients will not be evaluated if the arc length is significantly less then the resonant period and a shorter arc length decreases the significance of the gravitational model effects. The two day arc lengths of the GEOS I and GEOS II orbit reductions used for this analysis evaluate the lower degree coefficients but not the resonant harmonics. This is because the resonant harmonic periods for the GEOS I and GEOS II satellites are 7.1 and 6.3 days, respectively [7].

Table 5 shows the weighted root-mean-square (RMS) of the Doppler residuals for GEOS I and GEOS II orbit reductions using the different gravitational models. In these computations, the station coordinates were common for all of the reductions as well as the observational data of each satellite. These results show that the chopped and suppressed models yield approximately the same result. The increase in the weighted RMS for the SAO SE (II) adjusted model is more pronounced than for the similar WGS 72 model. Although these results are not conclusive, they do indicate that chopping is a better technique for truncation than

Table 5

Weighted RMS of Residuals
Using Various Gravitational Models
(Data Sigma=0.05 cy/sec)

Gravitational Model	Derivation		Weighted RMS	
	Method	Degree	GEOS I	GEOS II
WGS 72	Chopped	8	0.8043	0.4752
	Suppressed	8	0.8039	0.4784
	Adjusted	8	0.8161	0.5321
	Chopped	12	0.7838	0.4637
	Suppressed	12	0.7876	0.4583
	Adjusted	12	0.8313	0.4680
SAO SE (II)	Chopped	8	0.8019	0.5282
	Suppressed	8	0.8147	0.5327
	Adjusted	8	0.9049	0.6228
	Chopped	12	0.7929	0.5324
	Suppressed	12	0.7594	0.5470
	Adjusted	12	0.9513	0.7139

adjustment even though only two day data spans were used. The effect of truncation by any method would be even more pronounced with longer data spans due to the effect of neglecting the resonant harmonics (See Appendix).

Orbital differences were also computed by comparing the orbits reduced using the truncated models with those reduced using the complete model. The maximum and minimum orbital coordinate differences; radial, intrack, and crosstrack, along with the RMS differences are shown for the SAO SE (II) and WGS 72 gravitational models in Tables 7 and 8, respectively. In this coordinate system, the radial component lies along the geocentric radius to the satellite, the intrack component along the satellite velocity vector with the crosstrack component completing a right handed coordinate system. These results show that of the three techniques for truncation, the orbits reduced using the adjusted models produce the poorest agreement when compared to those produced using the complete models.

CONCLUSIONS

Comparison of the test results shows that the chopping of a more detailed gravitational model is preferred over the development of a limited model such as was done in the adjustment method. This is believed to be the result of the poor force modeling that occurs during the processing of satellite observational data if a zero value is assumed for the resonant harmonics. This concept is verified

Table 6

Orbital Differences in Meters Between Complete SAO SE (II) and SAO SE (II)
Trun[illegible]ted for Two Day Arc Reductions

Satellite	Method	Degree	Radial			Intrack			Crosstrack		
			Max	Min	RMS	Max	Min	RMS	Max	Min	RMS
GEOS I	Chopped	8	31.2	-24.8	9.6	60.8	-77.3	22.7	27.9	-26.9	8.7
	Suppressed	8	36.7	-29.6	11.4	71.6	-88.2	25.9	28.2	-30.5	9.5
	Adjusted	8	42.5	-36.5	14.2	116.3	-143.4	48.6	39.2	-39.5	15.1
	Chopped	12	15.5	- 7.8	5.7	38.6	-36.8	11.9	5.9	- 8.5	3.1
	Suppressed	12	35.2	-30.4	14.1	49.[illegible]	-47.7	16.0	17.7	-21.8	8.1
	Adjusted	12	22.2	-52.1	18.1	138.7	-[illegible]19.8	55.0	52.2	-42.5	19.7
GEOS II	Chopped	8	16.9	-10.6	6.4	57.4	-42.3	20.7	23.9	-26.8	9.7
	Suppressed	8	16.7	-10.1	6.3	57.1	-44.7	21.0	46.7	-46.9	19.8
	Adjusted	8	15.3	-43.5	16.2	100.8	-85.3	40.7	65.3	-57.4	23.5
	Chopped	12	12.1	- 8.0	4.8	28.8	-28.6	12.8	17.9	-18.2	7.0
	Suppressed	12	3.2	-20.5	12.0	27.4	-31.0	13.1	44.5	-48.9	18.9
	Adjusted	12	18.9	-40.2	16.5	116.8	-78.1	39.2	60.1	-64.3	24.0

Table 7

Orbital Differences in Meters Between Complete WGS 72 and WGS 72
Truncated for Two Day Arc Reductions

Satellite	Method	Degree	Radial			Intrack			Crosstrack		
			Max	Min	RMS	Max	Min	RMS	Max	Min	RMS
GEOS I	Chopped	8	21.8	-21.1	7.3	47.7	-63.5	19.7	20.4	-20.7	6.4
	Suppressed	8	22.8	-21.7	7.7	48.6	-66.7	20.2	18.2	-20.0	7.5
	Adjusted	8	19.2	-17.5	7.4	70.5	-59.5	25.0	22.7	-25.8	9.2
	Chopped	12	12.1	-11.3	4.5	32.9	-39.0	11.5	14.0	-13.6	4.3
	Suppressed	12	15.0	-13.0	5.5	37.3	-41.2	13.6	10.8	-10.7	4.1
	Adjusted	12	24.3	-21.3	9.4	68.1	-65.9	30.2	33.7	-32.9	13.2
GEOS II	Chopped	8	12.5	-13.5	5.1	56.2	-41.1	18.1	20.0	-19.5	7.2
	Suppressed	8	12.4	-13.2	5.3	53.1	-42.8	17.6	17.6	-26.1	7.5
	Adjusted	8	23.1	-15.8	7.1	60.7	-57.3	24.8	22.1	-26.5	9.0
	Chopped	12	7.0	- 7.4	3.1	39.6	-18.1	10.8	16.9	-17.9	6.5
	Suppressed	12	8.3	- 8.0	3.3	36.4	-20.8	10.9	14.8	-14.5	5.3
	Adjusted	12	19.9	-14.8	6.1	51.6	-29.8	17.5	27.0	-31.7	10.7

by the fact that the suppressed WGS 72 gravitational models with non-zero initial values for the resonant harmonic coefficients have signficantly lower degree variances and better surface gravity comparisons than their corresponding adjusted models.

The decision as to what model is best to use in an orbit reduction application is not as obvious. This is because of the fact that the significance of gravitational model errors varies with arc length and orbital characteristics. However, there is no real evidence to show that an adjusted model would be superior to a chopped model. Consequently, chopping of the WGS 72 gravitational model is recommended for applications in which a more complete model cannot be used.

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APPENDIX

ORBIT REDUCTIONS IN THE PRESENCE OF GRAVITATIONAL MODEL ERRORS

APPENDIX

The ideal gravitational model for orbit reductions is one that precisely describes the earth's gravitational field, Such a model would permit independence between the quality of the orbit reduction and arc length. This has not been achieved but is approached for a sample GEOS II arc using the WGS 72 gravitational coefficients. The weighted root-mean-squares of the tracking data residuals are 0.4773 and 0.4777 for two and eight day arc reductions, respectively. These almost identical root-mean-squares indicate that the WGS 72 coefficients producing significant perturbations on a GEOS II orbit, particualrly the 13th order resonant harmonics, are well defined. However, for the truncated gravitational models, the orbit errors are much larger for an eight day than for a two day arc length (Table A-1). This is caused by the eight day arc length exceeding the resonant period and thus preventing the secular appearance of the resonant intrack perturbations. When two day data fit spans are used, these perturbations have a secular rather than a periodic appearance and are thus partially absorbed into the orbital position and velocity vectors. Plots of the radial, intrack, and crosstrack orbital differences for the two day and eight day orbit reductions using gravitational models derived by chopping are shown in Figures A-1 through A-4.

Table A-1

Root-Mean Square Orbital Differences in Meters
Between Complete WGS 72 and WGS 72 Truncated
Gravitational Models for Two Day and Eight
Day GEOS II Arc Reductions

Gravitational Model	Arc Length	RMS		
		Radial	Intrack	Crosstrack
Chopped Eighth Degree	2 day	5.1	18.1	7.2
	8 day	10.3	131.1	11.3
Adjusted Eighth Degree	2 day	7.1	24.8	9.0
	8 day	11.1	130.8	11.1
Chopped 12th Degree	2 day	3.1	10.8	6.5
	8 day	11.4	130.7	12.7
Adjusted 12th Degree	2 day	6.1	17.5	10.7
	8 day	12.2	131.4	14.1

These results show that the selection of an appropriate data span is essential when truncated gravitational models are to be used.

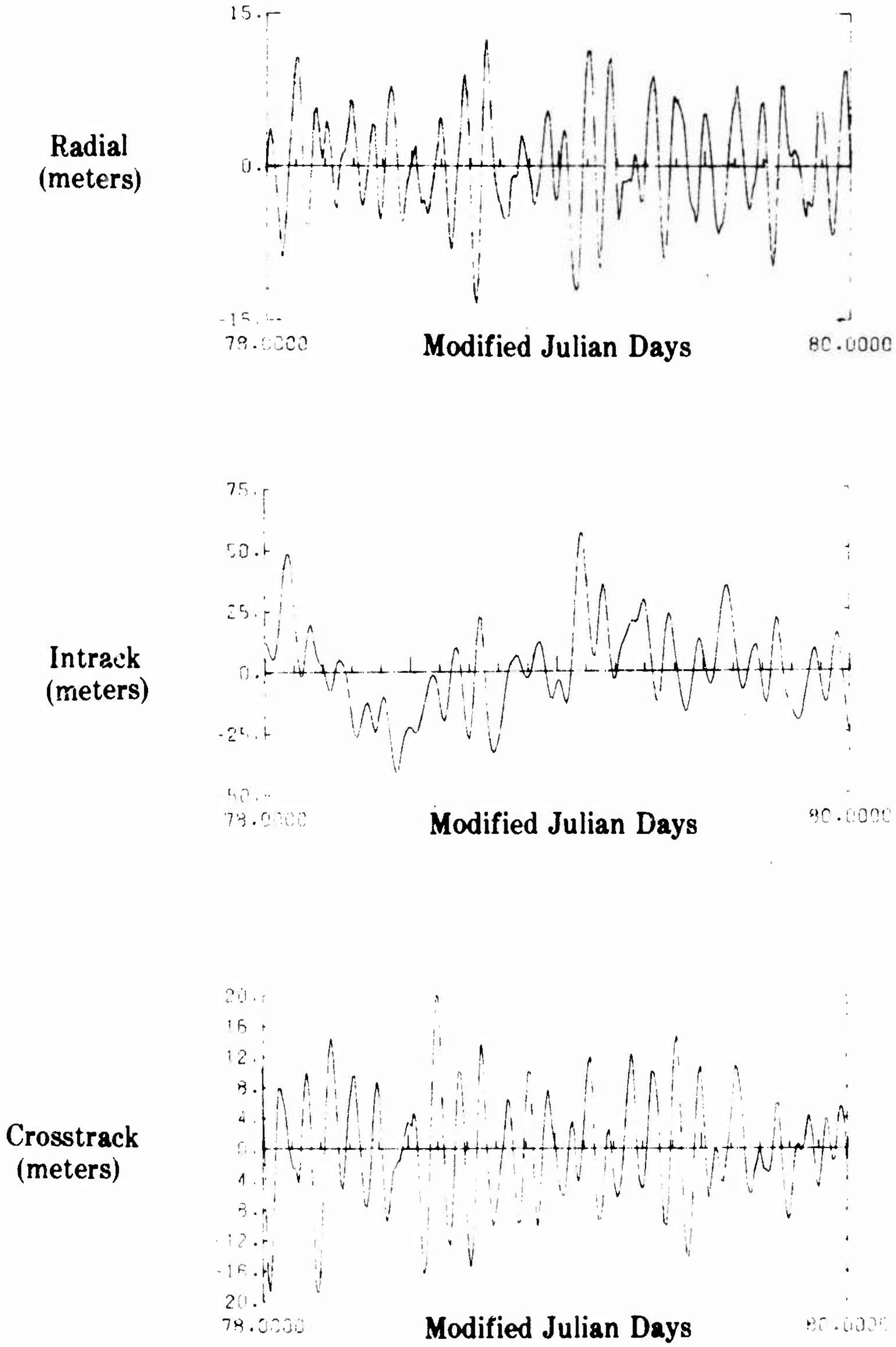

Figure A-1. Satellite Orbital Differences - WGS 72 Minus WGS 72 Chopped Eighth Degree - GEOS II Two Day Arcs.

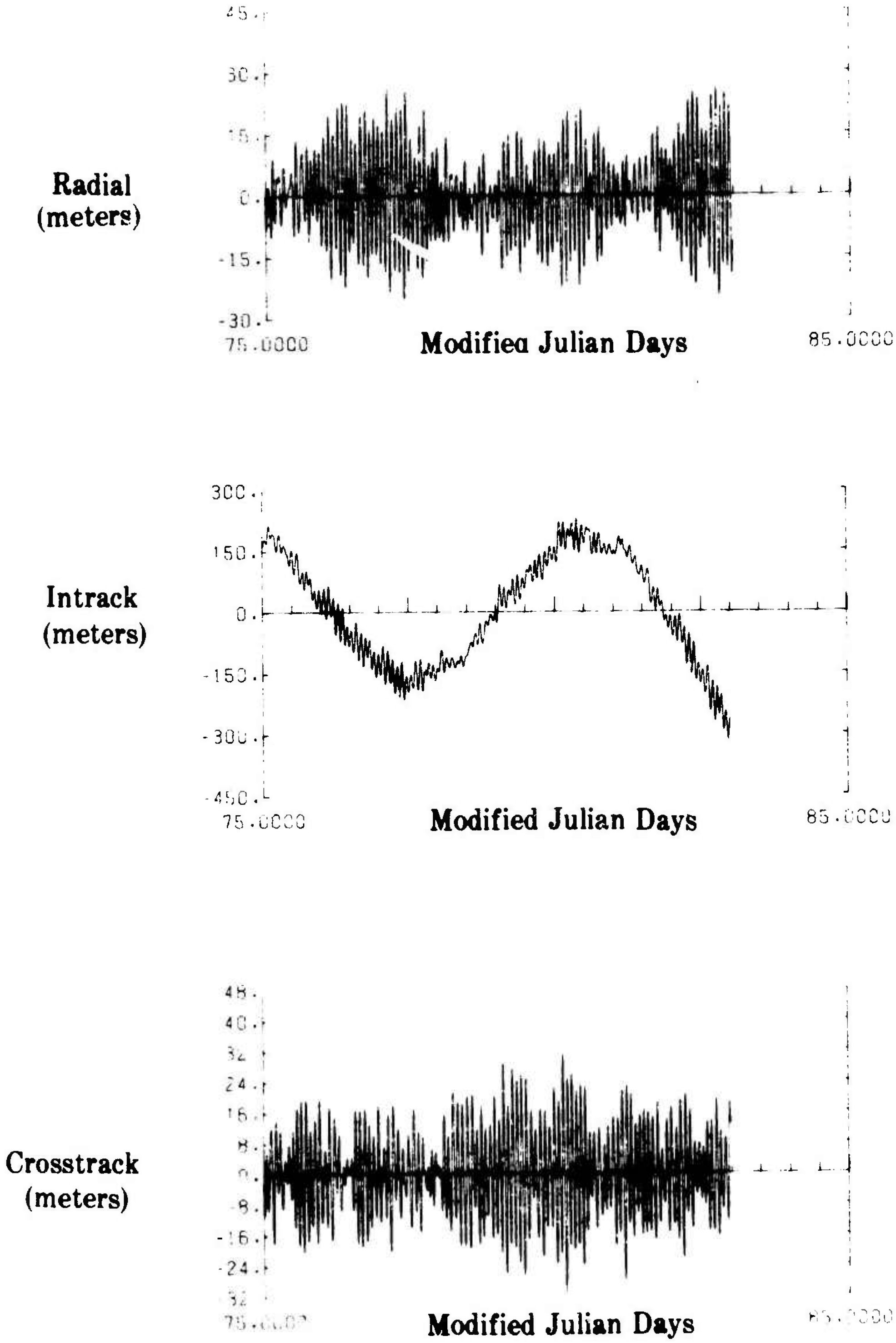

Figure A-2. Satellite Orbital Differences - WGS 72 Minus WGS 72 Chopped Eighth Degree - GEOS II Eight Day Arcs.

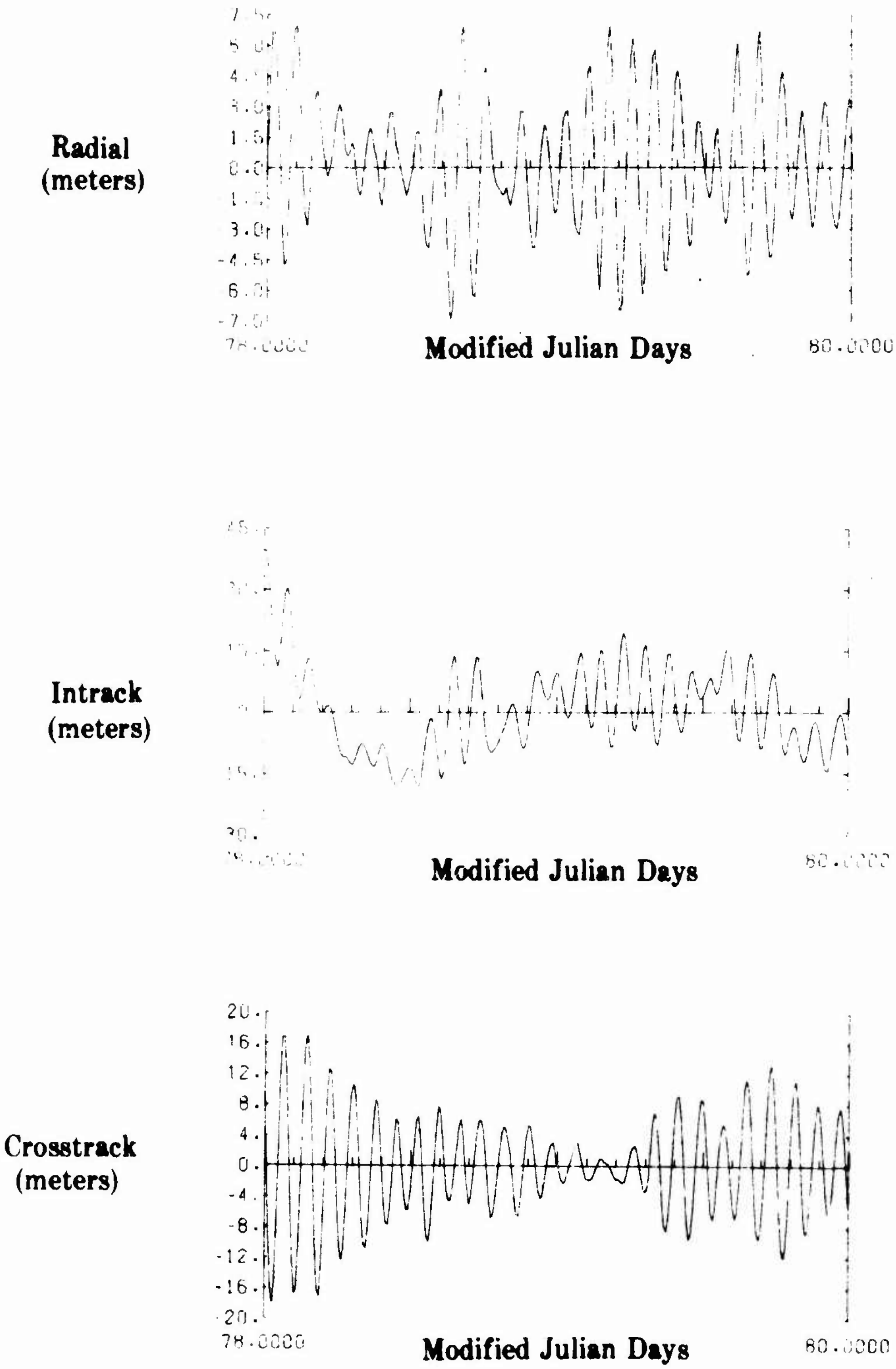

Figure A-3. **Satellite Orbital Differences-WGS 72 Minus WGS 72 Chopped 12th Degree-GEOS II Two Day Arcs.**

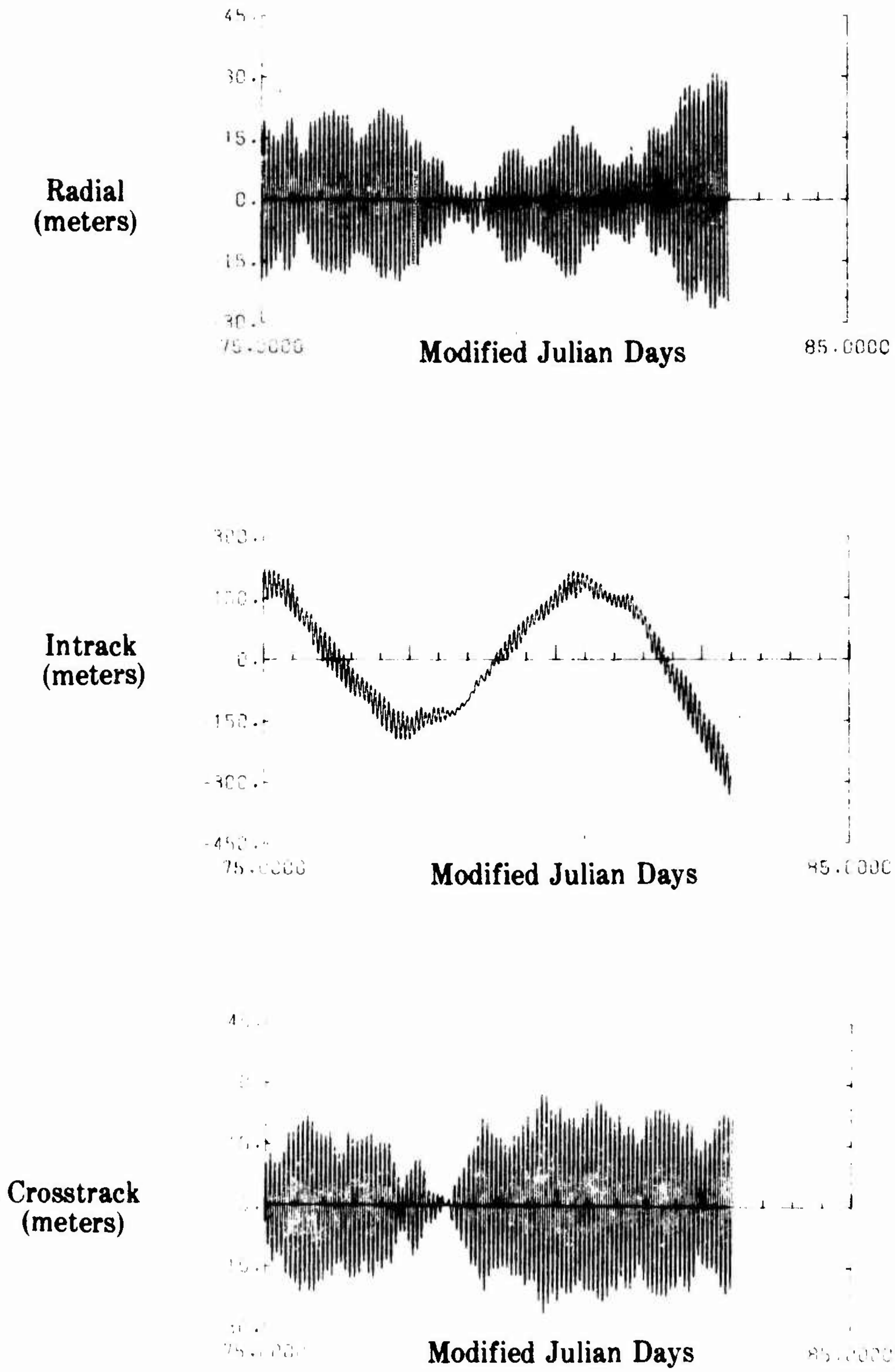

Figure A-4. Satellite Orbital Differences - WGS 72 Minus WGS 72 Chopped 12th Degree - GEOS II Eight Day Arcs.